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3D τ_{RR}-minimization in AdS$_4$ gauged supergravity

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Abstract: In this paper we propose the identification in AdS$_4$ $\mathcal{N}=2$ gauged supergravity of the coefficient τ_{RR} of 3D $\mathcal{N}=2$ SCFTs. We constrain the structure of this function in supergravity by combining the results from unitarity, holography and localization. We show that our conjectured function is minimized by the exact R-charge, corresponding to a gravitational attractor for the scalars of special geometry. We identify this mechanism with the supergravity dual of τ_{RR}-minimization. We check this proposal in ABJM model, comparing with expectations from localization and AdS/CFT duality. We comment also on possible relations with black hole microstates counting, recently obtained from application of localization techniques.

Keywords: AdS-CFT Correspondence, Field Theories in Lower Dimensions, Supersymmetric gauge theory

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Contents

1 Overview

The comprehension of the mathematical structure of renormalization group (RG) flow is one of the main goals of modern theoretical physics. A general expectation is that RG flows are irreversible: a theory's degrees of freedom reduce flowing from the ultraviolet (UV) to the infrared (IR) regime. There have been many proposals for quantifying this idea. In even dimensions exact results have been obtained with the aid of global anomalies; in particular, the existence of a monotonically decreasing function along the RG flow, interpolating between the UV and the IR fixed points, has been proven in 2D [1] and 4D [2]. In 2D such function coincides, at the UV and at the IR fixed points, with the Weyl anomaly, corresponding to the Virasoro central charge c. In 4D the same role is played by the coefficient of the Euler density, corresponding to the central charge a. These identifications led to the names of monotonicity c-theorem and a-theorem in 2D and 4D, respectively.

In supersymmetric field theories the central charge a has been computed non-perturbatively from the current correlators of the three point functions [3], and it is

$$a = \frac{3}{32}\left(\mathrm{Tr}R^3 - \mathrm{Tr}R\right) \tag{1.1}$$

where R is the charge associated to the global $U(1)_R$ symmetry. In the superconformal case the R-charge of an operator is related to its scaling dimension Δ, a consequence of the fact that the R-current is the lowest component of the stress tensor supermultiplet. The

R-symmetry current J_R is not, in general, the UV R-current J_0 since, along the RG flow, J_0 mixes with other global, abelian flavor symmetry currents F_i's. At the fixed point the exact R-current becomes

$$J_R = J_0 + \delta_i F^i \tag{1.2}$$

corresponding to the choice of δ_i coefficients[1] that maximize the central charge a [4]. This principle has been named a-maximization.

Superconformal field theories received significant attention for their central rôle in the holographic correspondence. Indeed, according to AdS/CFT, correlators of a strongly coupled d-dimensional CFT can be mapped to correlators at the boundary of a dual gravity theory in $d+1$ dimensions. Exploiting this correspondence has allowed the identification of the 4D SCFT central charge a with the superpotential of the dual $\mathcal{N}=2$ gauged supergravity theory admitting an AdS$_5$ vacuum [5]. In this framework, a key rôle is played by Chern-Simons coefficients of 5D gauged supergravity which, on the field theory side, are mapped holographically onto the anomalous coefficients of global currents three point functions. This observation, together with the identification of field theory R-charges with scalars of 5D, $\mathcal{N}=2$ supergravity very special geometry, allowed the reformulation of a-maximization in a supergravity language.

A similar formulation has been worked out in the last years for 3D SCFT. In this case, the setup is more complicated because of the absence of global anomalies, thus more involved techniques are necessary to test irreversibility of the RG flow. Moreover, the counterpart of a-maximization is not immediately identifiable.

This last problem was first solved in [6], where the coefficient C_T of the stress-energy tensor two point function has been used to determine the exact R-current. The quantity C_T is related by supersymmetry to the coefficient τ_{RR} of the R-current two point function, which has been shown to be minimized by the exact R-current (this result holds also in 4D as a corollary of a-maximization). Despite the simplicity of this relation the analysis of such quantity is difficult because it cannot be extracted from non perturbative analysis, making the method rather inefficient.

A breakthrough has, however, been achieved by applying localization techniques. Indeed, it has been shown that the free energy of the 4D field theory on S^3 (F_{S^3}) is extremized by the exact R-current [7]. By squashing the sphere it was proven that the free energy is in fact maximized by the exact R-charge, essentially for the same reason for which τ_{RR} is minimized [8]. At the RG flow fixed point the coefficient τ_{RR} is proportional to the free energy F_{S^3} [9], but a general functional relation in terms of R-charge mixing with the F_i's is not known.

The parallelism with the 4D case naturally turned the interest to the supergravity side. Calculation of the free energy F_{S^3} requires a holographic computation in Euclidean AdS$_4$ space, and the analysis of [10] has shown to correctly reproduce the large N behavior

[1]Observe that the J_0 coefficient depends on the R-current normalization; in this work we discuss canonical normalization. In general the difference between an R-current and a non-R-current is the fact that the supersymmetry generators Q's are charged under the former but not under the latter.

of ABJM model. More recently, a non perturbative analysis of the dual mechanism of localization in holography has been performed in [11].

The relation between the τ_{RR}-function and the AdS_4 case with Lorentzian signature has nevertheless been overlooked. It has been shown that one can obtain the τ_{RR}-function in AdS_4 gauged supergravity by consistent truncation of M-theory on SE_7 manifolds [12], but the approach requires the knowledge of the full 10D geometry.

In this paper we initiate the study of the τ_{RR}-function in $\mathcal{N}=2$ AdS_4 gauged supergravity, in presence of a generic (dyonic) gauging. We use the holographic dictionary to associate relevant field theory quantities, charges and currents, to the scalar fields and the gauge coupling in the 4D supergravity description. This dictionary allows us to propose the supergravity dual τ_{RR}-function. We show that its extremization corresponds to an attractor mechanism for the scalars of special geometry, while the minimization principle follows from a positivity requirement for the metric on the special Kähler manifold. Finally, the rôle of hypermultiplets is to further constrain the set of charges involved in the minimization. They provide a counterpart to the constraints imposed by superpotential interactions in the dual field theory.

The paper is organized as follows. In section 2 we review field theory aspects of τ_{RR}-minimization that will be useful for our discussion. We comment on the fixed point relation between the τ_{RR} function and F_{S^3} computed at large N. In section 3 we review basic aspects of gauged supergravity that will be relevant for our derivation of the holographic dual to τ_{RR}-minimization. Section 4 contains the main result of the paper. We identify the holographic dual of τ_{RR} function and show how a minimization principle arises from gauged supergravity. To do this it will be crucial to identify the R-current as the combination of vector fields in the gravitino variation, given by the symplectic product of holomorphic sections of special geometry with U(1) gauge fields. The exact R-current is the combination corresponding to the graviphoton of the $\mathcal{N}=2$ vacuum. By using special geometry constraints, the holographic dictionary and results from supersymmetric localization we identify the supergravity dual τ_{RR} with the quartic power of the superpotential W appearing in the gravitino variation. In section 5 we discuss the ABJM model to show our proposal at work. In section 6 we compare the off-shell behavior[2] of the τ_{RR}-function for ABJM model and expected from AdS/CFT duality. We also discuss the possibility of a large-N, off-shell relation between the function τ_{RR} and the free energy, in terms of a generic assignment of R-charges. In section 7 we emphasize the possible connection with microstates counting for AdS_4 black holes (that have $\mathrm{AdS}_2 \times S^2$ near-horizon geometry) involving a 1D R-charge extremization principle. We summarize the results and conclude with possible extensions of our work in section 8. Appendix A contains further details on AdS_4 gauged supergravity.

2 τ_{RR}-minimization

In this section we review the minimization principle as discussed in [6].

τ_{RR}-minimization is a method that allows to identify the exact R-charges assignement in a superconformal field theory among all the possible choices allowed by the supercon-

[2] By off-shell we mean before minimizing the τ_{RR}-function with respect to the mixing parameters.

formal algebra. The proof of this statement relies on the analysis of correlation functions of two point global currents in superconformal field theories:

$$\langle j_i^\mu(x) j_j^\nu(y)\rangle = \frac{\tau_{ij}}{(2\pi)^3}(\partial^2\delta^{\mu\nu} - \partial^{\mu\nu})\frac{1}{(x-y)^2} \tag{2.1}$$

where, because of unitarity, the matrix τ_{ij} has positive eigenvalues. In superconformal field theories, one of these global currents corresponds to the lowest component of the supermultiplet having the stress energy tensor as highest component, and it is referred to as the R-current.

In general, this current is a combination of the UV R-current R_0 and other flavor currents F_i. Defining a trial R-current

$$R_t = R_0 + \delta_i F_i \tag{2.2}$$

the exact R-current corresponds to a specific assignment of the mixing coefficients δ_i. Some combinations in (2.2) are usually excluded by the structure of the interactions but this is not enough, in general, to fix the coefficients δ_i. A closer look at the correlation functions allows to identify the necessary constraints. The coefficient $\tau_{R_t R_t}$ is

$$\tau_{R_t R_t} = \tau_{R_0 R_0} + 2\sum \delta_i \tau_{R_0 i} + \sum_{i,j}\delta_i\delta_j\tau_{ij} \tag{2.3}$$

As a function of the mixing coefficients, (2.3) is minimized by the choice of δ_i corresponding to the exact R-current. This has been proven by studying the first and the second derivatives of τ_{RR} in the δ_i space, which are precisely $\tau_{R_0 i}$ and τ_{ij} coefficients. The first is set to zero by supersymmetry which then imposes an extremization condition. Minimization finally follows by the unitarity property of the matrix τ_{ij}.

Despite the simplicity of this result, τ_{RR} extremization did not become a popular method to extract the exact R-charge, because of the absence of anomalies in 3D. The general, non-perturbative structure of τ_{RR} is not known and this function can be used to compute the exact R-charge only at weak coupling in the perturbative expansion. A more tractable object for computation is the free energy F_{S^3}, obtained from localization of the path integral on S^3 [7, 13, 14], which gives a non-perturbative, exact result. It has been shown that F_{S^3} is maximized by the exact R-charge [8]. Moreover, the non perturbative nature of this function allowed the conjecture of a 3D F-theorem [16], subsequently proven in [15]. On the contrary, counterexamples to a monotonic behavior of τ_{RR} during the RG flow have been found in [17].

At the fixed point of supersymmetric field theories, in the large N limit, F_{S^3} and τ_{RR} are simply related as[3]

$$F_{S^3}^{\max} = \frac{\pi}{2}\tau_{RR}^{\min} \tag{2.4}$$

In the rest of the paper we look for the τ_{RR}-function and its minimization principle from the point of view of $\mathcal{N}=2$ AdS$_4$ gauged supergravity.

[3]Observe that in our conventions this relation differs from the one obtained in [9] by an overall factor $\pi/2$.

3 AdS$_4$ gauged supergravity

In this section we review some general aspects of AdS$_4$ gauged supergravity, in order to introduce relevant quantities and fix necessary notations for the rest of the analysis. We provide further details in appendix A.

At the supergravity vacuum all fermionic fields are set to zero and consistently decouple from the bosonic equations of motion. In order to find an anti de Sitter vacuum, the relevant dynamics is described by the bosonic part of the action. In $\mathcal{N}=2$ Supergravity, coupled to n_V vector multiplets and n_H hypermultiplets, it can be written as

$$S = \int d^4x \Big(-\frac{R}{2} + \mathcal{I}_{\Lambda\Sigma} F^{\Lambda}_{\mu\nu} F^{\Sigma\,\mu\nu} + \frac{1}{2\sqrt{-g}} \mathcal{R}_{\Lambda\Sigma} \epsilon^{\mu\nu\rho\sigma} F^{\Lambda}_{\mu\nu} F^{\Sigma}_{\rho\sigma} + \\ + g_{i\bar{j}} \partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}} + h_{uv} \nabla_\mu q^u \nabla^\mu q^v - V_g(z,\bar{z},q) \Big) \tag{3.1}$$

Our study deals with abelian gaugings, as a consequence only scalars in the hypermultiplets are charged under the gauge fields, while scalars in the vector multiplets remain neutral. It corresponds to the request that only isometries of the quaternionic manifolds are gauged, namely that the potential is of the form

$$V_g(z,\bar{z},q) = 4 h_{uv} \langle k^u(q),\, \mathcal{V}(z,\bar{z})\rangle \, \langle k^u(q),\, \overline{\mathcal{V}}(z,\bar{z})\rangle - 3 W\overline{W} + g^{i\bar{j}} D_i W D_{\bar{j}} \overline{W} \tag{3.2}$$

We are introducing here a complex function built from the product of the moment maps with the $(2n_V+2)$-dimensional symplectic sections as[4]

$$W(z,\bar{z},q) = \langle \mathcal{P}^x(q),\, \mathcal{V}(z,\bar{z})\rangle \equiv e^{K/2} \left(\mathcal{P}^x_\Lambda X^\Lambda - \mathcal{P}^{x\Lambda} F_\Lambda \right) \tag{3.4}$$

This reduces to the domain walls superpotential in the case of U(1) R-symmetry gauging (Fayet-Iliopoulos). For a complete account of definitions of special geometry and gauged supergravity we refer to the appendix.

A configuration with zero fermions and zero gauge fields is supersymmetric if the corresponding supersymmetry variations vanish. They involve the scalar fields and are explicitly given by[5]

$$\begin{aligned} \delta\psi^A_\mu &= D_\mu \epsilon^A - \frac{1}{2} (\sigma^x)^A_B \gamma_\mu W \epsilon^B + \dots \\ \delta\lambda^{iA} &= i g^{i\bar{j}} (\sigma^x)^A_B D_{\bar{j}} \overline{W} \epsilon^B + \dots \\ \delta\zeta^\alpha &= \mathcal{U}^A_{u\alpha} \langle k^u, \overline{\mathcal{V}} \rangle \epsilon_A + \dots \end{aligned} \tag{3.5}$$

where the dots indicate terms which are identically zero at the vacuum. In all the cases we consider we are always able to use SU(2) symmetry to rotate the moment maps $\mathcal{P}^x$

[4] We will generically consider both electric and magnetic gauging, thus we indicate

$$\mathcal{P}^x = \mathcal{P}^x_\lambda(\Theta^{\Lambda\lambda}, \Theta^\lambda_\Lambda)\,, \tag{3.3}$$

for a generic choice of embedding tensor Θ [18]. Analogously, the Killing vectors will be given in a symplectic vector $k^u = (k^{u\Lambda}(q), k^u_\Lambda(q))$.

[5] The supersymmetric transformations of bosonic fields are identically zero at the vacuum since they are proportional to fermionic fields.

in the direction $\mathcal{P}^3$, with $\mathcal{P}^1 = \mathcal{P}^2 = 0$ (in particular this is how definition (3.4) has to be read). Notice that there are cases where this is not possible [19, 20], and this requires modifications in the analysis. We will leave this point to further investigations.

The conditions for the supersymmetric vacuum are then

$$\partial_i |W|\,\big|_{\{q^{u*},\, z^{i*},\, \bar{z}^{i*}\}} = 0\,, \qquad \langle k^u(q^*), e^{-K/2}\mathcal{V}(z^*)\rangle = 0 \tag{3.6}$$

where the extremization is done only over the scalars z^i of the Special Kähler manifold, and $\{q^{u*},\, z^{i*},\, \bar{z}^{i*}\}$ indicate the value of the scalar fields at the minimum. The second condition depends on how many Killing vectors are identically zero at the vacuum. In particular, the non zero Killing vectors corresponds to a number n_c of algebraic, holomorphic constraints on the fields z^i, related to the Higgsing of n_c abelian vector fields at the vacuum [21]. Moreover, because of special geometry, if the supersymmetric vacuum is given by the scalar configuration $(q^*, z^*, \bar{z}^*)$, the AdS_4 radius, and thus the cosmological constant at the minimum of the potential can be expressed as

$$-\frac{\Lambda}{3} = \frac{1}{\ell^2_{\rm AdS}} = -\frac{1}{2}\mathcal{P}^{x\,T}(q^*)\mathcal{M}(z^*,\bar{z}^*)\mathcal{P}^x(q^*) \tag{3.7}$$

by the scalar dependent $\mathrm{Sp}(2n_V+2)$ matrix

$$\mathcal{M}(z^i,\bar{z}^{\bar{i}}) = \begin{pmatrix} \mathcal{I}+\mathcal{R}\mathcal{I}^{-1}\mathcal{R} & -\mathcal{R}\mathcal{I}^{-1} \\ -\mathcal{I}^{-1}\mathcal{R} & \mathcal{I}^{-1} \end{pmatrix} \tag{3.8}$$

The matrices $\mathcal{I} \equiv \mathcal{I}_{\Lambda\Sigma}$ and $\mathcal{R} \equiv \mathcal{R}_{\Lambda\Sigma}$ gives the coupling between scalars and gauge fields in the Lagrangian (3.1). It is important to notice that at the extremum (3.6), the inverse of AdS length square is given by the square root of the quartic symplectic invariant $\mathcal{I}_4(\mathcal{G})$ [22, 23] valued at the "charges" given by the moment maps evaluated at the vacuum as

$$\mathcal{G} \equiv \mathcal{P}^x(q^*)\,. \tag{3.9}$$

This follows immediately from the study of black hole horizons attractors [24–26], where exactly the same extremization occurs with respect to the scalars of the vector multiplets z^i. In that case the charges $\mathcal{G}$ are the black hole charges and the quartic invariant gives the value of the black hole entropy, or better, the volume of the S^2 at the horizon (black hole area).

4 τ_{RR} in $\mathcal{N} = 2$ gauged supergravity

In this section we study τ_{RR}-minimization from gauged supergravity. We start our analysis with the identification of conserved currents and the charges that determine their mixing. We notice that there can be other broken global currents, in general, but we ignore this possibility in the first part of the discussion. Namely, we first restrict our analysis to the case in which the second equation in (3.6) is solved by setting k^u to zero. This corresponds of restricting to the sector of conserved currents that mix with the R-charge, with constant moment maps.

The photons in the supersymmetric variations (3.5) are identified with the R and the conserved global currents of the dual field theory. In general, photons appear in the supersymmetric variations of the fermions combined with the superpotential. In the case of the R-current the combination is proportional to the superpotential while in the case of the gaugino there is also a derivative involved. This distinguishes the R current from other conserved global currents. In the dual field theory this translates into the fact that the supercharges are charged under the R-current while they are neutral under the non-R globally conserved currents. The coefficients of the mixing can be read from the variation of the gravitino. In general they are proportional to the symplectic sections $\mathcal{V}$ that parameterize the superpotential as in (3.4). In formulas, by referring to a symplectic vector of R-charges as s, we have $s = t\mathcal{V}$.

The normalization coefficient[6] is imposed by fixing the charge of the gravitino to 1. This choice corresponds to $t = 1/W$, as it can be read from the supersymmetric variation of ψ_μ^A. Our R-charges become

$$s = \frac{\mathcal{V}}{W} \tag{4.1}$$

The flavor currents are obtained by acting with derivatives on $\mathcal{V}$, and the combinations of the charges that determines the exact R-current is given by (3.6).

At this point we can try to identify the τ_{RR}-function in AdS_4 gauged supergravity. The starting point is the observation that the coefficient of flavor currents two point functions is dual, in the AdS/CFT correspondence, to the inverse square of Yang Mills coupling. We then adapt the discussion of [12] to the symplectic invariant formalism, pertaining to $\mathcal{N} = 2$ supergravity. In this case we use the matrix $\mathcal{M}$ introduced in formula (3.8) and consier the expression

$$\tau_{RR} = \mathcal{T}\, s^T \mathcal{M} \bar{s} \tag{4.2}$$

We observe that this is a real function, depending on a combination of complex R-charges. This notion requires some interpretations. In an electric gauging we can restrict to the simpler formula $\tau_{RR} \propto \mathcal{I}_{\Lambda\Sigma} s^\Lambda \bar{s}^\Sigma$, where one can set the imaginary parts of the sections to zero and treat the R-charges as real. An analogous discussion holds for magnetic gauging, however the situation is more delicate for dyonic gauging, where one should apply a symplectic rotation to identify the correct combinations leading to real R-charges. As discussed in section 2 dyonic gaugings can thus be more subtle [19, 20] and deserve further investigations.

We proceed by inserting the R-charges (4.1) in (4.2), simplifying the expression by using special geometry constraints. We obtain

$$\tau_{RR} = \mathcal{T}\, \mathcal{M} s \bar{s} = \frac{\mathcal{T}}{|W|^2}\,. \tag{4.3}$$

The extremization condition corresponds to the requirement of an $\mathcal{N} = 2$ AdS vacuum (3.6). The signs of the second derivatives are imposed from the constraints of the

[6] One can choose also other normalizations, it is just important to consider this difference when mapping the normalization of the charges described in supergravity with the ones on the field theory side.

special geometry, and by the positivity of the scalar metric in (A.2). Here we obtain

$$\partial_i \partial_{\bar{j}} |W|^{-2} \propto -g_{i\bar{j}} |W|^{-2} \,. \tag{4.4}$$

This relation does not seem to be correct, because it would lead to a maximization of τ_{RR} instead of a minimization.

Let's discuss a possible resolution of this problem, similarly to the discussion of [12], where τ_{RR} was computed from AdS/CFT correspondence. We refer the reader to the discussion on section 6 for clarification on the resolution of the puzzle, as proposed here, and to the proposal of [12] for a similar problem. The holographic dictionary allows to derive the expression of two point functions at the fixed point, but leaves an ambiguity in the behavior of such correlators in terms of the mixing parameters. A useful way to fix this ambiguity consists of considering a specific case (e.g. the ABJM model) in which the relation is known and, by comparison with the relation obtained via τ_{RR}-minimization, fully calibrate the functional dependence of τ_{RR} on the mixing coefficients.

In order to provide a resolution, then, we first observe that, at the fixed point, the function τ_{RR} is reproduced by[7]

$$\tau_{RR} = \mathcal{T} \frac{|W|^{p-2}}{\mathcal{I}_4^{p/4}} \tag{4.5}$$

The extremal value of this function and its first derivative are independent from p. We notice that the Hessian at the extremum $\partial_i |W| = 0$ can be obtained by using (A.8) and it is given by

$$\partial_{\bar{j}} \partial_i \tau_{RR} = g_{i\bar{j}} (p-2) \tau_{RR} \,. \tag{4.6}$$

Different values of p are consistent with the extremization of the conjectured τ_{RR}-function but they can lead to a maximization instead of a minimization problems. One can then speculate on the meaning of the different values of p. In the holographic dictionary, in generic dimensions d, the coefficient τ_{RR} is a function of the AdS length scale, $\ell_{\mathrm{AdS}}^{d-3}$, so for $d = 3$ the dependence of τ_{RR} on ℓ_{AdS} drops out. We notice also that, when studying the functional behavior of the function τ_{RR}, we take into account its dependence on the scalars in the vector multiplets while keeping the moment maps constant (i.e. the hyperscalars are fixed at their supersymmetric vacuum values). In view of this observation, we interpret the relation (3.7) as reinserting the AdS scale in the analysis, i.e. introducing another dynamical object in the minimization problem. Although this does not modify the dictionary explained above, it can modify the derivatives and the off-shell behavior of τ_{RR}.

Another possibility for mismatch resides in the identification of the R-charges. The graviphoton field of the supergravity theory can be read from the gravitino variation and, in a purely electric gauging, is given by $e^{\mathcal{K}/2} X^\Lambda \mathcal{I}_{\Lambda\Sigma} A_\mu^\Sigma$. The R charges in this case are simply $s^\Lambda = X^\Lambda/(X^\Lambda P_\Lambda)$. The explicit structure of the graviphoton is then obtained from the relation above, by evaluating $\mathcal{I}_{\Lambda\Sigma}$ at the fixed point, where this gives the correct mixing

[7] In the following discussion we consider the quartic invariant $\mathcal{I}_4^{p/4}$ as the AdS scale, i.e. we fix its value at the supersymmetric vacuum, and it represents a fixed scale of the theory. On the other hand, we consider the superpotential W as a function of the constrained scalars in the special geometry. In the field theory interpretation the former is constant while the latter is a function of the R-charges.

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of the abelian charges. Out of the fixed point, however, the matrix $\mathcal{I}_{\Lambda\Sigma}$ is a function of the sections and the charge mixing is in general not known. This can be another source of problems in the identification of τ_{RR} in (4.3). In the rest of the analysis we will fix $\mathcal{I}_{\Lambda\Sigma}$ to their vacuum value, when identifying the charges of the photons. It would be interesting to come back to this problem in the future.

In the rest of the analysis we fix the coefficient of p above to match the off shell relation that one can guess from the AdS/CFT analysis of [12]. Here we propose that the correct power is $p = 6$, for now it is a conjectural choice and we will be more concrete on such relation in section 6. The τ_{RR}-function becomes

$$\tau_{RR} = \mathcal{T}\frac{|W|^4}{\mathcal{I}_4^{3/2}} \tag{4.7}$$

Observe that this discussion is valid in absence of magnetic fluxes, we will comment on their role in section 7. Moreover, eq. (4.7) suggests that the functional dependence of τ_{RR} on the charges is $\tau_{RR} \propto (s^T \mathcal{M}\bar{s})^{-2}$.

This concludes our identification of the supergravity dual of τ_{RR}-minimization principle discussed in [12]. Furthermore, we can fix the proportionality constant $\mathcal{T}$ using (2.4) and the relation $|W^{\min}|^4 = \mathcal{I}_4$. We obtain

$$\tau_{RR}^{\min} = \mathcal{T}\frac{|W^{\min}|^4}{\mathcal{I}_4^{3/2}} = \frac{2}{\pi}F_{S^3}^{\max} \quad \to \quad \mathcal{T} = \frac{2}{\pi}F_{S^3}^{\max}\sqrt{\mathcal{I}_4} \tag{4.8}$$

By imposing this normalization our candidate supergravity dual τ_{RR}-function can be written as

$$\tau_{RR} = \frac{2}{\pi}F_{S^3}^{\max}\frac{|W|^4}{\mathcal{I}_4} \tag{4.9}$$

So far we considered only $\mathcal{N} = 2$ AdS_4 gauged supergravity in presence of vector multiplets, fixing the hypermultiplets to their supersymmetric vacuum. We used them only to define the moment maps necessary to identify the mixing of the currents. Let us now briefly discuss their rôle in the extremization of τ_{RR}. Dynamical hypermultiplet scalars are related to the presence of massive gauge bosons in the holographic dual theory, i.e. to broken global symmetries on the field theory side.

Let us first take a closer look at the supersymmetric vacuum equations. In the discussion above we only considered the solution to (3.6) given by $k^u = 0$. By expanding around this solution, we now allow some $k^u \neq 0$, and keep the linear order in the hypermultiplet scalars. This splits $\mathcal{M}_H$ it two parts, $\mathcal{M}_{H_1}$ and $\mathcal{M}_{H_2}$, defined as follows. For the $\mathcal{M}_{H_1}$ submanifold the situation corresponds to the one described above, namely k^u are vanishing and they do not provide further constraints on $\mathcal{V}$. The submanifold $\mathcal{M}_{H_2}$ corresponds to n_{H_2} non vanishing k^u, signaling spontaneous breaking of the gauge group. There are then $n_{H_2} \leq n_V$ massive gauge bosons that acquire a mass by an Higgs mechanism, eating n_{H_2} scalars in $\mathcal{M}_{H_2}$ and leaving only $3n_{H_2}$ scalars on $\mathcal{M}_{H_2}$. The moment maps $\mathcal{P}^x$ become functions of the uneaten hyperscalars. Here, as discussed above, we can use still an SU(2) transformation when expanding around the vacuum, and consider only one non vanishing component, $\mathcal{P}^3$, depending on n_{H_2} coordinates on $\mathcal{M}_{H_2}$. This is possible if the Killing

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potentials are expanded around the supersymmetric vacuum at linear order in the hyperscalars [5]. At the same time, the solution of the hyperino variation for the non vanishing components of k^u imposes n_{H_2} conditions on the scalars of $\mathcal{M}_V$. They are exactly the n_{H_2} constraints imposed by the n_{H_2} hyperscalars in $\mathcal{P}^3$.

This discussion is very similar the one performed in [5] in the case of AdS_5 gauged supergravity, and it has an interesting connection with the results obtained on the field theory side in [27, 28]. We remark here that in 4D SCFTs the effect of the broken symmetries is captured in the context of a-maximization by including Lagrange multipliers in the extremization procedure. The multipliers impose the superpotential (and the anomaly) constraints and they are associated to the coupling constants. This procedure actually matched the perturbative results in field theory, i.e. one can expand the exact R-charges in terms of the multipliers and one finds a match with the perturbative expansion. This led to identify the multipliers of the field theory description with the hyperscalars in the moment map $\mathcal{P}^3$ enforcing the constraints on $\mathcal{V}$.

It is tempting to give a similar interpretation to the hypermultiplets as Lagrange multipliers for the 3D τ_{RR} function. However, observe that in 3D there are not constraints coming from global anomalies. Nevertheless, there are superpotential couplings breaking some of the global symmetries that could mix with the R-charge.[8] One may wonder if the effect of these couplings can be captured by the τ_{RR} function. In 3D a similar proposal for τ_{RR} is missing, but it has been shown that the multipliers can be considered in the extremization problem of F_{S^3} [29], where a two loop matching was observed. This allows us to propose a similar mechanism also in the context of τ_{RR}-minimization and motivates our search of the supergravity dual of the Lagrange multipliers.

The presence of the multipliers on the supergravity side allows also the study of R-symmetric RG flows, along the lines of [30][9] (see also [32] for another discussion on the role of the multipliers in the AdS_5 case).

5 The τ_{RR} function of the ABJM model

In this section we apply our formalism to the calculation of the holographic τ_{RR}-function for the ABJM model.

In general the simplest example that one can consider consists of $\mathcal{N} = 2$ gauged supergravity with a graviton multiplet and n_V vector multiplets. Here we choose the case with $n_V = 3$ and focus on a solution discussed in [33]. This theory corresponds to a consistent truncation of S^7, and it accounts for a deformation of the ABJM model, along the Cartan $U(1)^4$ of the $SO(8)_R$ symmetry group. The model admits a formulation in terms of a prepotential

$$\mathcal{F} = -2\sqrt{-X^0X^1X^2X^3}\,. \tag{5.1}$$

[8] Observe that the discussion is not generic and it applies to cases in which there is a weakly Higgsed vector multiplet in the bulk rather that any SCFT with superpotential. We thank the referee for comments on this issue.

[9] In may be interesting to study the RG flow connecting two AdS_4 vacua in gauged supergravity obtained in [31].

The symplectic vector $\mathcal{V}$ and the Kähler potential $\mathcal{K}$ are

$$\mathcal{V} = e^{\mathcal{K}/2}(1, t_2t_3, t_1t_3, t_1t_2, -it_1t_2t_3, -it_1, -it_2, -it_3), \quad e^{-\mathcal{K}} = 8Re(t_1)Re(t_2)Re(t_3)\,. \quad (5.2)$$

We consider a purely electric gauging, with charges $g_i = \frac{1}{2}$ $(i = 1, \ldots, 4)$. The moment maps are constant functions of these charges and can be rotated to a single nonzero component $\mathcal{P}^3$ given by

$$\mathcal{P}^3 = \frac{1}{2}(0, 0, 0, 0; 1, 1, 1, 1)\,. \quad (5.3)$$

The τ_{RR}-function in this case is

$$\tau_{RR} = \frac{F_{S^3}^{\max}}{64\pi^2} \frac{|1 + t_1t_2 + t_1t_3 + t_2t_3|^4}{(\mathrm{Re}(t_1)\,\mathrm{Re}(t_2)\,\mathrm{Re}(t_3))^2} \quad (5.4)$$

where $F_{S^3}^{\max}$ corresponds to the maximal value of the F_{S^3}, used here as an input obtained with localization techniques. At the extremal point the scalars t_i are fixed to their values at the $\mathcal{N} = 2$ supersymmetric vacuum: $t_i = 1$. This is the supersymmetric attractor mechanism for AdS_4 corresponding to the condition (3.6). In the field theory interpretation it represents an extremization condition on the R-charges (4.1). The τ_{RR} function evaluated at the minimum matches the expected result (2.4).

We can also compute the R-charges. We notice that, in presence of an electric gauging, the graviphoton can be written in terms of the sections as $e^{\mathcal{K}/2}X^\Lambda F_\Lambda^{\mu\nu}$. As discussed the R-charges refer to the field strength $\mathcal{I}_{\Lambda\Sigma}F^{\Sigma\mu\nu} \equiv F_\Lambda^{\mu\nu}$, treating $\mathcal{I}_{\Lambda\Sigma}$ as a constant matrix, fixed at the AdS_4 value. The R-charges for this model are chosen as $s^\Lambda = e^{\mathcal{K}/2}X^\Lambda/2\mathcal{W}$, and we can focus to the case of real scalars. Thus, by turning off the imaginary parts of the coordinates $t_i = b_i + iv_i$, we parameterize the R-charges as

$$s^1 = \frac{2}{B} \qquad s^2 = \frac{2b_2b_3}{B} \qquad s^3 = \frac{2b_1b_3}{B} \qquad s^4 = \frac{2b_1b_2}{B} \quad (5.5)$$

where we defined $B = 1 + \sum_{i<j} b_ib_j$. At the fixed point the scalars are $b_i = 1$ and the R-charges s^Λ are all equal to $\frac{1}{2}$. At $k = 1$ the normalization $\tau_{RR}^{\min}$ can be extracted from [16] and we find

$$\tau_{RR}^{\min} = \frac{2}{\pi}F_{S^3}^{\max} = \frac{4\sqrt{2}}{3\pi}N^{3/2}\,. \quad (5.6)$$

Observe that in this case we can describe the function τ_{RR} before the extremization as a function of the R-charges s^Λ. We have

$$\tau_{RR} = \frac{\sqrt{2}}{3\pi}N^{3/2}\frac{1}{16\, s^1s^2s^3s^4}\,. \quad (5.7)$$

It is interesting to compare this result with the expectations from localization since, in this example, it is possible to associate the R-charges in supegravity with those of the ABJM model. The supergravity model can be interpreted indeed as a consistent truncation of a deformed ABJM model. On the field theory side, the ABJM model corresponds to a 3D quiver gauge theory with $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ gauge groups, where k is an integer Chern-Simons (CS) level. There are two pairs of bifundamental fields a_i and b_j with superpotential

$$W = a_1b_1a_2b_2 - a_1b_2a_2b_1\,. \quad (5.8)$$

The theory has $\mathcal{N}=6$ supersymmetry enhanced to $\mathcal{N}=8$ for $k=1,2$. The model that we considered in this section, corresponding to set the FI parameters of the gauging $g_i=1/2$, is a consistent truncation of the ABJM model preserving the full Cartan $\mathrm{U}(1)^4$ symmetry. The situation with generic g_i corresponds to the topological twist discussed in [34]. We will come back to this in section 7.

The free energy on the field theory side can be parameterized by the R-charges of the four fields a_1, a_2, b_1 and b_2. The general parameterization respecting the $\mathrm{U}(1)^4$ Cartan symmetry is

$$\begin{aligned} \Delta_{a_1} &= \delta_0+\delta_1+\delta_2+\delta_3, & \Delta_{a_2} &= \delta_0+\delta_1-\delta_2-\delta_3 \\ \Delta_{b_1} &= \delta_0-\delta_1+\delta_2-\delta_3, & \Delta_{b_2} &= \delta_0-\delta_1-\delta_2+\delta_3 \end{aligned} \tag{5.9}$$

The free energy at large N depends on the R-charges as

$$F_{S^3}=\frac{\sqrt{2}\pi}{3}N^{3/2}\sqrt{\Delta_1\Delta_2\Delta_3\Delta_4}\,, \tag{5.10}$$

and its maximization fixes $\Delta_{a_i}=\Delta_{b_i}=\frac{1}{2}$. The exact R-current is obtained by the combination[10]

$$R_{ex}=\delta_0 J_0+\delta_i F_i \tag{5.11}$$

where F_i represent the three $\mathrm{U}(1)$ currents that can mix with the R-charge. In this case, at the fixed point we have δ_i=0, thus the canonically normalized, exact R-current corresponds to $J_0/2$. This is an expected result, because in this case two of the global $\mathrm{U}(1)$s are actually in the Cartan of the global $\mathrm{SU}(2)^2$ symmetry group and the other $\mathrm{U}(1)$ is a baryonic symmetry.

We want to compare this with the supergravity result obtained in (5.7) for τ_{RR}. In the example at hand the graviphoton field is given by the combination $s^\Lambda A^\mu_\Lambda$, where s^Λ are the charges parameterized in (5.5). The relation between the s^Λ and the δ_i variables is

$$\begin{aligned} \delta_0 &= \frac{s^0+s^1+s^2+s^3}{2}, & \delta_1 &= \frac{s^0+s^1-s^2-s^3}{2} \\ \delta_2 &= \frac{s^0-s^1+s^2-s^3}{2}, & \delta_3 &= \frac{s^0-s^1-s^2+s^3}{2} \end{aligned} \tag{5.12}$$

supplemented by the constraint $s^\Lambda P_\Lambda=1$, that here translates to $\sum s^\Lambda=2$. At the vacuum this reproduces the expected relation $\delta_0=1$ and $\delta_i=0$.

To conclude this section, let us compare the functional behavior of the functions τ_{RR} and $F^2_{S^3}$, in terms of the parameterization found in (5.12), which gives: $F^2_{S^3}(s^\Lambda)\propto\tau^{-1}_{RR}(s^\Lambda)$. This relation is the one expected from AdS/CFT correspondence, as we will comment in section 6, and it provides a consistency check of our conjecture on the structure of τ_{RR}.

[10] Here we refer to the canonical normalization of the R symmetry $R_0=(d-2)/2J_0$, this explains the difference in the normalization discussed in (2.2).

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6 Relation with the large N F_{S^3} and $Vol(SE_7)$

In this section we further comment on the relation between our results and predictions from supersymmetric localization, we also conjecture the structure of τ_{RR} compatible with a functional relation between τ_{RR} and F_{S^3} in terms of the R-charges. Such a relation is expected from the results of [12], taking into account the dependence of τ_{RR} on the volume form $Vol(Y)$, appearing in supergravity effective description of M-theory compactified on a SE_7 manifold "Y".

In general F_{S^3} and τ_{RR} have different functional dependence on a generic R-charges assignment. Nevertheless, as we observed in the case of the ABJM theory, our conjectured definition of τ_{RR} leads to the functional relation $\tau_{RR} \propto F_{S^3}^{-2}$, once the R-charges on the two sides of the duality are identified. This corresponds in fact to the choice $p = 6$ in (4.5).

Let us briefly review the results of [12]. In AdS/CFT, the volume $Vol(Y)$ is parameterized by the components of the Reeb vector $\mathbf{b}$, which is a Killing vector for one of the U(1) isometries of Y, that corresponds to the R-symmetry U(1) in the dual field theory. The exact R-charge is obtained by minimizing the volumes with respect to the components of $\mathbf{b}$ [35]. It was observed that at the fixed point τ_{RR}, obtained from the Kaluza-Klein reduction on the volume formula, is proportional to the inverse volume. This led to an apparent contradiction, being both the functions[11] $\tau_{RR}(\mathbf{b})$ and $Vol_{\mathbf{b}}(Y)$ minimized by the exact R-charge. The way-out proposed in [12] was to distinguish a functional dependence of $\tau_{RR}(\mathbf{b})$ from $Vol_{\mathbf{b}}(Y)$ and a normalization to respect of the volume at the fixed point, $Vol_{\min}(Y)$. In this way, it still make sense to have two different principles of minimization for $\tau_{RR}(\mathbf{b})$ and $Vol_{\mathbf{b}}(Y)$ but an inverse functional dependence from the R-charge parametrized by the components of the vector $\mathbf{b}$. In this paper we observed a similar mechanism at work in gauged supergravity. The relation between τ_{RR} and the volume is [12]

$$\tau_{RR}(\mathbf{b}) = \frac{4\pi^2}{3\sqrt{6}} \left(\frac{N}{Vol_{\min}(Y)} \right)^{3/2} Vol_{\mathbf{b}}(Y_7)\,, \tag{6.1}$$

however, the general relation between the free energy and the volume $Vol(Y)$ is

$$F_{S^3}(\mathbf{b}) = N^{3/2} \sqrt{\frac{2\pi^6}{27 Vol_{\mathbf{b}}(Y)}}\,. \tag{6.2}$$

By combining the two relations (6.2) and (6.1) one obtains

$$\tau_{RR} = \frac{2}{\pi} \frac{(F_{S^3}^{\max})^3}{F_{S^3}^2} \tag{6.3}$$

where $F_{S^3}^{\max}$ is the maximized free energy corresponding to $Vol_{\min}(Y)$ in (6.1), and this predicts a relation between τ_{RR} and the large N free energy F_{S^3}.

We observe that the relation (6.3) is valid at large N for a generic quiver gauge theory describing a stack of N M2 branes probing the tip of a toric Calabi-Yau (CY) fourfolds.

[11] Here we refer to the τ_{RR}-function computed from the AdS/CFT correspondence. For this reason we express the dependence from the components of the Reeb vector $\mathbf{b}$.

It would be interesting to study if it can be extended to other classes of theories, for example quiver gauge theories which are conjectured to be dual to massive type IIA backgrounds [19, 20, 51].

We finally notice that we fixed $p = 6$ in (4.5) to match the predictions on τ_{RR} for the ABJM model. As we discussed above it would be important to have a more direct derivation of this result from gauged supergravity, which would also shed light on a general proposal, based on the analogy with the 4D case [36]. The expectation is that the volume form can always be associated to a quartic function of the R-charges [37, 38], and hints towards the validity of this proposal can be read in results of [39]. We hope to come back to these questions in the future.

7 Topological twist and relation with AdS_2 BH entropy

In this section we discuss possible relations with a very interesting result, recently appeared in [34]. In that paper the authors provided the counting of black hole microstates for solutions in asymptotically AdS_4, reproducing the Bekenstein-Hawking (BH) entropy from the calculation of an index in the holographic dual gauge theory. The index is obtained as a function of the R-charges of the dual superconformal field theory, and the black hole entropy matches the index when the latter is evaluated for the exact R-charges. To obtain the exact R-charge the authors exploited an extremization principle dual, on the gravity side, to an attractor mechanism. Since the field theory lives in an odd dimensional space-time, they proposed an extremization mechanism in analogy with the maximization of F_{S^3}, observing that the Witten index on S^1 has indeed the desired properties. Having now a proposal for τ_{RR}, one could use our results to obtain extremization properties analogous to those of F_{S^3} in that paper. We observe that it is possible to give an analogous formulation by studying the holographic τ_{RR}-function on $\mathrm{AdS}_2 \times S^2$. When compactifying AdS_4 on $\mathrm{AdS}_2 \times S^2$ with magnetic fluxes turned on, the AdS_4 superpotential W_4 reduces to the ratio of the AdS_2 central charge Z_{2D} and the AdS_2 superpotential W_{2D}.

As we observed above, a generic function proportional to the superpotential $|W|$ is extremized by the exact R-charge. Nevertheless, magnetic fluxes can mix with the R-charge and, as a consequence, different functions have different extremization properties. It seems reasonable to study the function at $p = 2$ in (4.5). This is maximized by the exact R-charge also in the case when the theory is deformed by the magnetic fluxes. Moreover, along the gravitational flow, this function becomes [40]

$$\tau_{RR} = \frac{2}{\pi G_4 |W_4|^2} \to \frac{1}{G_4}\left|\frac{Z_2}{W_2}\right| . \tag{7.1}$$

This reduction corresponds to a dynamical running of scalars fields from AdS_4 to $\mathrm{AdS}_2 \times S^2$: their mixing induces a flow to a different attractor point. In field theory language, adding magnetic fluxes is equivalent to perform a topological twist on the flavor symmetries, and the new attractor can be reformulated by a different mixing of the R-current with the fluxes in the dual 1D superconformal quantum mechanics. In this case, the AdS_2 attractor equation fixes the correct mixing and should correspond to an R-charge extremization principle on the field theory side. At the vacuum, eq. (7.1) becomes

$$\frac{1}{G_4}\left|\frac{Z_2}{W_2}\right| \propto \frac{R_{S^2}^2}{G_4} \propto S_{BH}\,, \tag{7.2}$$

reproducing the BH entropy.

It would be interesting to investigate further in this direction. For example, one could apply our minimization mechanism to reproduce and extend the results of [34] for other BPS black holes, e.g. those obtained in [41]. Our analysis suggests, in fact, that the transformation of (7.1) may be captured by an R-charge extremization principle, at work in the topologically twisted field theory dual to the 4D gauged supergravity.

8 Conclusive discussion: open problems and further investigations

In this paper we studied the coefficient of the two point function for the R-current of 3D $\mathcal{N} = 2$ SCFTs from $\mathcal{N} = 2$ AdS$_4$ gauged supergravity. By exploiting special geometry constraints we have conjectured the supergravity dual of τ_{RR}-function (4.9). We have derived the extremization principle of [12] in a gravitational setup, to obtain the exact mixing of the R-current with abelian symmetries. We have found that it corresponds to an attractor mechanism for the scalars in the vector multiplets. We discussed the role of the quaternionic manifold, motivating how the hypermultiplet scalars can be interpreted as Lagrange multipliers, constraining the extremization. Our analysis does not require the existence of a prepotential and it applies for various choices of gauging, thus different setups.

In our derivation we conjectured the behavior of τ_{RR} in order to reproduce the AdS/CFT predictions in the case of the ABJM model. In this way, we've obtained a relation between the τ_{RR}-function and the free energy F_{S^3} for general R-charges, matching previous expectations relying on volume computations. More general checks and studies in this direction are necessary. Indeed, we did not consider here other truncations that could be of interest for the AdS/CFT correspondence. Among those, we mention the dual theories conjectured in [42–44] for the truncation of M^{111} and Q^{111}, where one could compare our results with the predictions from the volume formula obtained from a geometric analysis. In this context, the goal would be to identify the general behavior of τ_{RR} in gauged supergravity in terms of the Reeb vector, for a general truncation of M-theory on a SE$_7$ manifolds, along the lines of [45]. We observe that, for some of these theories, the calculation of the free energy does not reproduce an $N^{3/2}$ scaling behavior [16]. Our analysis in gauged supergravity would thus provide a different holographic check for these models.

We also discussed a possible relation between our construction and the results of [34]. The results in that paper motivates a direct study of the R-charge extremization problem from the 1D perspective, following an analogous approach to the ones developed in [46–49] compactifying from AdS$_5$ to AdS$_3 \times S^2$.

In our analysis we only analyzed cases without higher derivatives terms, whose contribution would correspond to models with non vanishing TrR. Another analysis deserving further investigation is the study of the global properties of the hyperscalar manifold. This is necessary for studying flows between supersymmetric solutions. Here, by including the

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effects of the Lagrange multipliers, we restricted to the possibility of R-symmetric supersymmetric RG flows.

We conclude with a final observation. The prepotential $\mathcal{F} = C_{IJK}X^I X^J X^K/X^0$ corresponds to the very special Kähler geometry and it is related to the AdS_5 supergravity setup. In fact, it can be obtained by reducing a 5D $\mathcal{N}=2$ gauged supergravity with very special geometry defined by the same C_{IJK} tensor. It would be interesting to investigate if this plays any role in the relation between the free energy and the central charge obtained in [50], where an interpolation between 4D a-maximization and 3D F-maximization was obtained. Recently, another connection between a and F was discussed in [51], but in that case our analysis may get modified by the presence of a dyonic gauging. One may try to connect these result and the special geometry of the AdS_5 and the AdS_4 supergravity.

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A Definitions and useful identities of gauged $\mathcal{N}=2$ supergravity

A general $N=2$ theory[12] can be coupled to n_V vector multiplets $(A^I_\mu, \lambda^{iA}, \lambda^{i*}_A, z^i)$, containing complex scalar fields z^i $(I, i = 1, \dots, n_V)$, and n_H hypermultiplets (ζ^α, q^u), containing real scalars $(\alpha = 1, \dots, 2n_H,\ u = 1, \dots, 4n_H)$.

The $4n_H$ real q^u scalars are in fact coordinates of a quaternionic manifold $\mathcal{QM}$ of quaternionic dimension n_H. The choice of gauging considered in this work involves a group of isometries $G \in \mathcal{QM}$. It is defined by a set of moment maps $\mathcal{P}^x(q)$ related to the Killing vectors as [21]

$$-2k^u_\Lambda K^x_u v = \nabla_v \mathcal{P}^x_\lambda \,, \tag{A.1}$$

where K^x is the curvature of the SU(2) connection on the quaternionic manifold.

The complex scalars z^i parametrize a special Kähler manifold $\mathcal{SM}$, whose geometry is completely defined by a Kähler potential $K(z,\bar{z})$, from which the metric of the manifold is derived as

$$g_{i\bar{j}}(z,\bar{z}) = \partial_i \partial_{\bar{j}} K(z,\bar{z}) \,. \tag{A.2}$$

It is convenient to parametrize the special Kähler scalar fields with holomorphic symplectic sections of a projective bundle, $(X^\Lambda(z), F_\Lambda(z))^T$, $\Lambda = 0, 1, \dots, n_V$, satisfying (bar indicates complex conjugation)

$$F_\Lambda \bar{X}^\Lambda - X^\Lambda \bar{F}_\Lambda = -ie^{-K} \,. \tag{A.3}$$

[12]Definitions and conventions used in the paper are explained in this appendix. For a complete discussion of $N=2$ gauged supergravity we refer to the review [52]. For a more general analysis on gauged $N=2$ vacua we refer to [21].

The expression above defines the symplectic product as

$$\langle A_1, A_2 \rangle = A_1^T \Omega A_2 \,, \qquad \Omega = \begin{pmatrix} 0 & I_{2n_V+2} \\ -I_{2n_V+2} & 0 \end{pmatrix} \,, \tag{A.4}$$

on any $\mathrm{Sp}(2n_v + 2)$ vector $A = (A_\Lambda, A^\Lambda)$. The normalized symplectic sections are then

$$\mathcal{V} = e^{K/2}(X^\Lambda(z), F_\Lambda(z))^T \,, \qquad \langle \mathcal{V}, \bar{\mathcal{V}} \rangle = -i \,, \tag{A.5}$$

they satisfy

$$\begin{aligned} D_i \mathcal{V} &= \partial_i \mathcal{V} + \frac{1}{2} \partial_i K \, \mathcal{V} \,, & D_{\bar{i}} \mathcal{V} &= \partial_{\bar{i}} \mathcal{V} - \frac{1}{2} \partial_{\bar{i}} K \, \mathcal{V} = 0 \,, \\ D_{\bar{i}} \bar{\mathcal{V}} &= \partial_{\bar{i}} \bar{\mathcal{V}} + \frac{1}{2} \partial_{\bar{i}} K \, \bar{\mathcal{V}} \,, & D_i \bar{\mathcal{V}} &= \partial_i \bar{\mathcal{V}} - \frac{1}{2} \partial_i K \, \bar{\mathcal{V}} = 0 \,. \end{aligned} \tag{A.6}$$

By using the special geometry identity

$$D_{\bar{j}} D_i \mathcal{V} = g_{i\bar{j}} \mathcal{V} \,, \tag{A.7}$$

one can derive the following relations used in section 4

$$\begin{aligned} \frac{2\partial_{\bar{j}} \partial_i |W|}{|W|} \Big|_{\partial_i |W| = 0} &= g_{i\bar{j}} \big|_{\partial_i |W| = 0} \,, \\ \partial_{\bar{j}} \partial_i |W|^{p-2} \big|_{\partial_i |W| = 0} &= g_{i\bar{j}} (p-2) |W|^{p-2} \big|_{\partial_i |W| = 0} \,. \end{aligned} \tag{A.8}$$

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